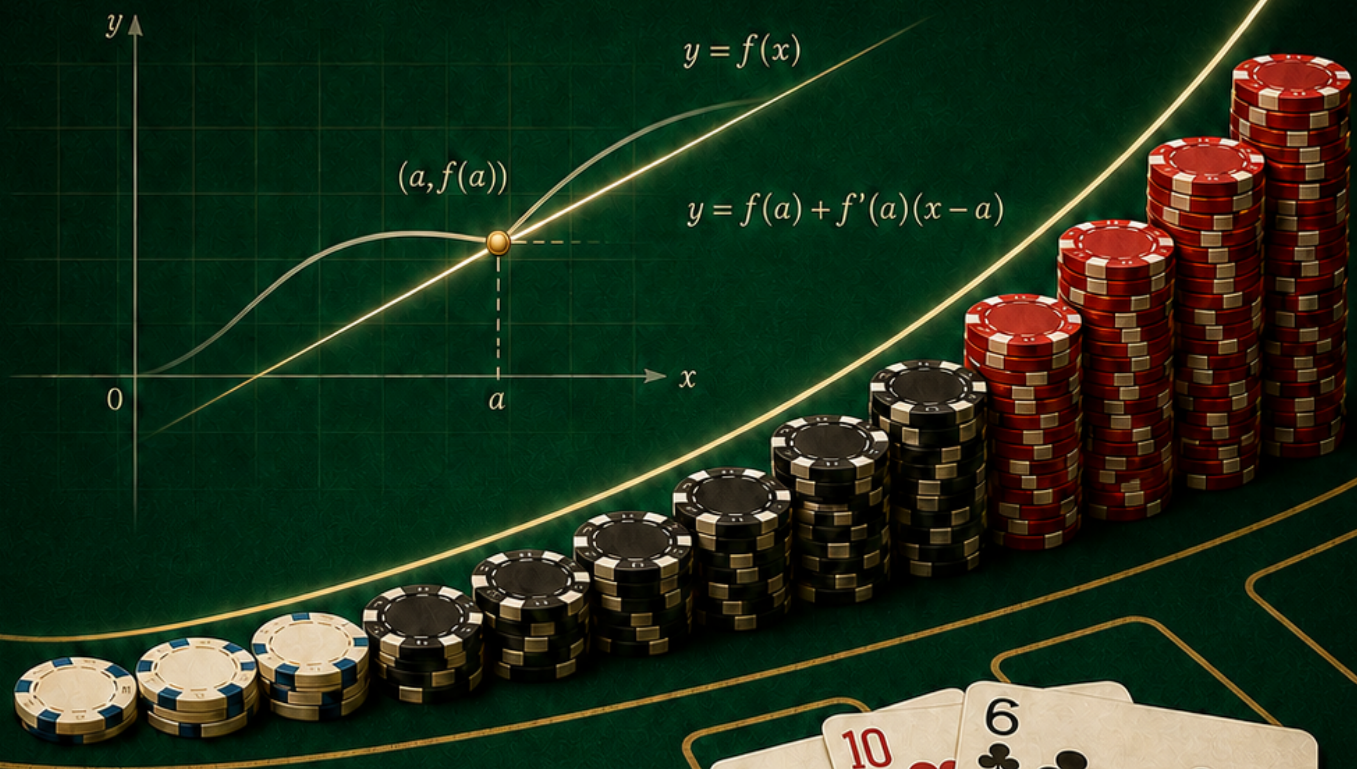
Calculus I

At the Blackjack Table





Limits • Derivatives • Integrals

ISAIAH SENDY

THE SAMPLE

Calculus I at the Blackjack Table

A first look at the book: front matter and Chapter 1

Limits • Derivatives • Integrals

ISAIAH SENDY

BEFORE THE CHAPTER

What This Sample Includes

This sample is a reader-facing preview, not a production blueprint. It includes a short orientation, a clear promise for the book, a quick guide to the blackjack ideas used in the examples, and the full first chapter.

1	The Book Promise	What the course is trying to make possible.
2	How to Read This Book	How the blackjack context supports, but never replaces, the mathematics.
3	Two-Minute Table Guide	Only the card-play vocabulary needed to begin.
4	Chapter 1	Functions, domains, graphs, transformations, composition, and inverse rules.

THE NORTH STAR

The Book Promise

> *You will learn single-variable calculus by watching how small changes become rules, decisions, rates, accumulated effects, and long-run outcomes.*

Blackjack gives the book a recurring physical setting: chips move, a count changes, a payout rule changes, and a model succeeds only when its assumptions are honest. The point is not to glorify gambling. The point is to give abstract mathematics a table you can see.

When the game clarifies the idea, we use it. When plain mathematics teaches faster, we leave the table. That balance matters.

ORIENTATION

How to Read This Book

Each lesson is built around one mathematical move. Sometimes that move is shown at the table; sometimes it is shown with a clean symbolic example. The recurring blackjack setting is there to make the math concrete, not to decorate the page.

What to watch for

- Rules: which inputs are allowed, and what output is promised.
- Change: how a graph, formula, or model responds when one quantity moves.
- Honesty: where an average model is useful, and where variance can fool the eye.
- Transfer: moments where the same calculus idea leaves the table entirely.

The blackjack examples assume chips as units, not dollars. That keeps the mathematics focused on structure: inputs, outputs, slopes, areas, and accumulated change.

FOR READERS NEW TO THE GAME

A Two-Minute Guide to the Table

You do not need to play blackjack to read this book. You only need a few ideas. A player tries to beat the dealer without going over 21. A natural blackjack is an ace plus a ten-value card on the first two cards, and a traditional table pays 3-to-2 on that hand.

The count is a way of tracking which kinds of cards have appeared. High cards left in the shoe tend to help the player; low cards left tend to help the house. The true count adjusts that running count by the number of decks remaining.

Blackjack idea	Calculus home
A bet becomes a payout	Function rule
A minimum or maximum bet	Domain restriction
The running count changes by cards seen	Stepwise change and accumulation
A true count divides by decks left	Average value or density
A payout changes from 3-to-2 to 6-to-5	Slope and transformation
A cash-out converts chips to dollars	Composition and inverse rules

THE COURSE MAP

Where the Book Goes

The full book is planned as a complete Calculus I path. This sample starts with functions because every later topic needs a dependable language for inputs, outputs, and change.

Part I	Foundations through Differentiation	functions, limits, continuity, derivatives
Part II	Curves, Approximation, and Integration	curve sketching, optimization, antiderivatives, sums
Part III	Accumulation and the Fundamental Theorem	integrals, net change, average value, area
Capstone	The Calculus of a Shoe	one sustained model from shuffle to cut card

Chapter 1 begins now: functions as rules that change.

CHAPTER

1

Functions: Rules That Change

You are sitting at third base. The dealer slides you an ace and a king. Four chips are in the betting circle; six chips come back. The table did not improvise. Your bet entered a rule, and the rule returned an answer. **That rule is a function.**

Calculus is the mathematics of change: how one quantity moves when another does. Before we can measure change, we need a clean way to say what depends on what. That language is the *function*, and every later chapter depends on it.

Blackjack gives us honest examples because the table is full of rules: a bet turns into a payout, a count turns into a wager, a shoe changes one card at a time. We will use those examples when they sharpen the idea and step away from the table whenever plain mathematics is clearer. All bets and payouts are measured in *chips*, not dollars.

LESSON 1

A Rule You Can Trust

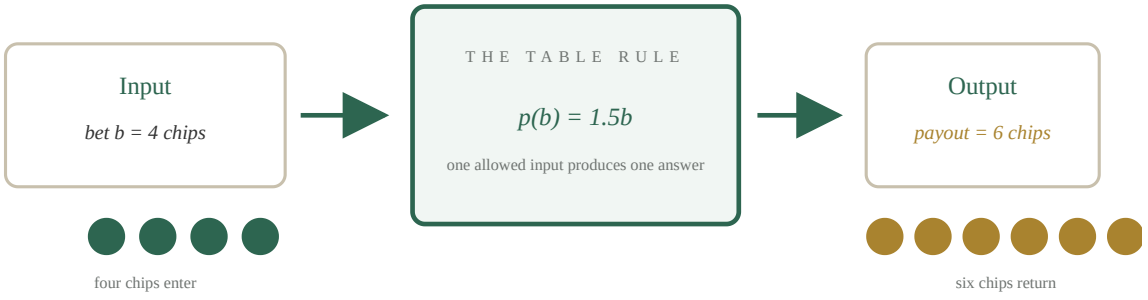
A function is a rule with a promise: each allowed input gives exactly one output. The promise is what makes the rule useful. If the same input can lead to several different answers, the rule may describe a real situation, but it is not yet a function.

Start with a clean table rule. On a 3-to-2 blackjack table, a natural blackjack pays one and a half times the bet. If b is the bet in chips and $p(b)$ is the blackjack payout, then

$$p(b) = 1.5b$$

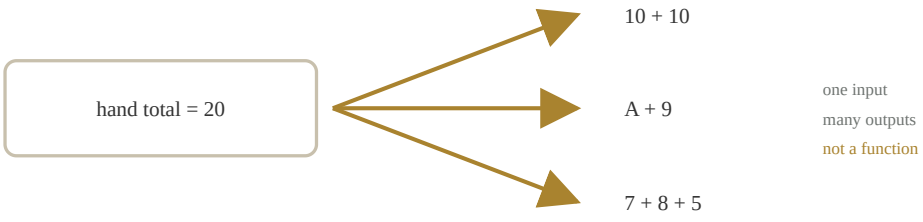
Feed the rule 4 chips and it returns 6. Feed it 20 and it returns 30. The rule does not care who is playing, whether the last hand was lucky, or how noisy the table is. Same input, same output.

- **TABLE MOMENT** *You slide four chips into the circle. The dealer flips a ten. "Blackjack. Pays six." Same bet, same answer, every time. That is the promise.*



A function is a dependable rule: one input, one rule, one output.

The phrase *exactly one output* is doing real work. "Hand total 20" can come from 10+10, A+9, or 7+8+5. If the input is only "total 20" and the output is "the cards," the promise breaks. One input has several outputs.



One total can be made several ways, so "cards from total" is not a function.

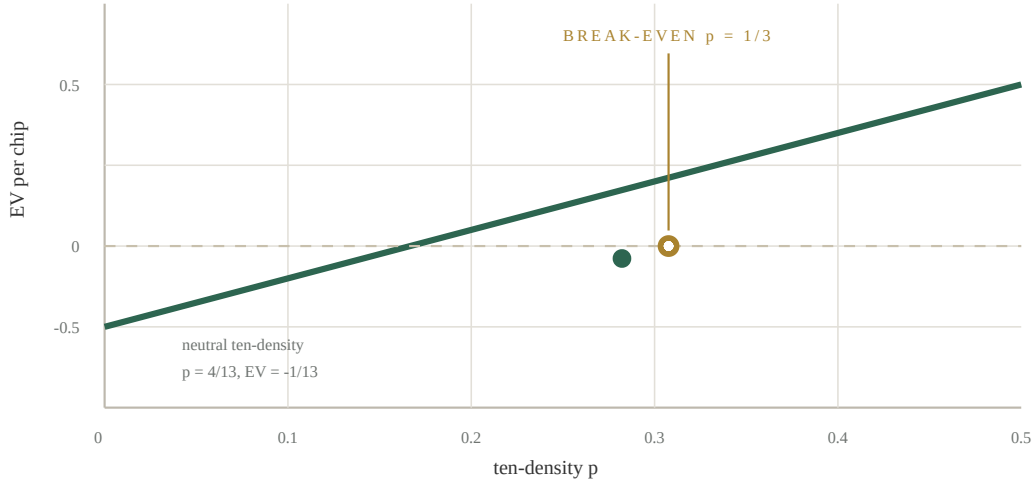
Outputs do not have to be numbers. Basic strategy is a function too: the input is a complete situation - your hand, the dealer upcard, whether the hand is soft, and the available options - and the output is a decision such as hit, stand, double, split, or surrender. The point is not that the output is numerical. The point is that the input is specific enough to force one answer.

A SECOND FUNCTION: INSURANCE

When the dealer shows an ace, insurance is a side bet that pays 2 to 1 if the hole card is a ten-value card. For a one-chip insurance bet, let p be the fraction of unseen cards that are tens. Then the expected profit per chip risked is

$$EV(p) = 3p - 1$$

This model assumes the only information we use is the ten-density p. At a neutral composition, p is about 4/13, giving EV(4/13) = -1/13. At p = 1/3, the bet breaks even.



Insurance is a line: below p = 1/3 it loses value; above p = 1/3 it gains value.

THIS IS NOT REALLY ABOUT BLACKJACK

The table gives the idea a body, but the structure is much larger than the game. Celsius turns into Fahrenheit by the function $F(c) = 1.8c + 32$. A phone bill might turn gigabytes into cost by $C(g) = 30 + 5g$. Same skeleton, different costume. A function is useful because it lets you move from a situation to a rule and from a rule to a prediction.

That is why the input must be named carefully. "A ten-chip blackjack bet" is not enough to predict profit after a hand, because the hand could win, lose, push, or pay a natural. But "a ten-chip natural blackjack at a 3-to-2 table" is specific enough. Good modeling is often the art of making the input precise.

WHERE THIS POINTS IN CALCULUS

Calculus studies how an output changes as its input moves. That question only makes sense if each input has a single, definite output to change from. The plain definition in this lesson is the ground under limits, slopes, derivatives, and integrals.

PRACTICE

BUILD Write the payout function for a 6-to-5 table. What does a 25-chip blackjack pay?

DECIDE A rule pays 10 chips at a true count of 1 or lower, then 10 more for each count above that. Is it a function?

EVALUATE Using EV(p) = 3p - 1, find the insurance expected value at p = 0.30 and p = 0.40.

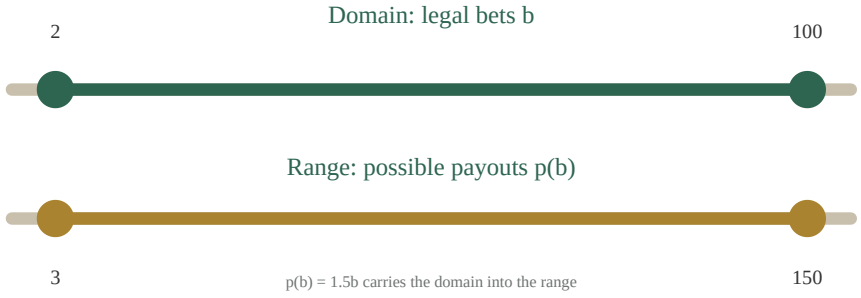
LESSON 2

Which Inputs Are Allowed

A rule is only half the story. The other half is the set of inputs it is allowed to accept. A function that works for some numbers and not for others is only honest if it says so out loud.

The payout rule $p(b) = 1.5b$ looks as if it accepts any number. A real table does not. If the placard says minimum 2 chips and maximum 100 chips, then the allowed bets form the **domain**: every bet from 2 through 100. Once the domain is fixed, the **range** follows: payouts from 3 through 150.

- **TABLE MOMENT** *The felt says 2 to 100. Try to bet one chip and the dealer pushes it back. Domain is not a suggestion.*



Domain and range are connected: allowed inputs determine reachable outputs.

Sometimes the domain is limited by the arithmetic itself. The rule $w(b) = 30/b$ reports chips won per chip risked. It can accept any positive bet, but it cannot accept $b = 0$. Dividing by zero has no meaning, so the mathematics throws that input out whether or not the casino would allow it.

A DOMAIN WITH A HARD WALL

A running count is also a function, but its domain is set by the shoe. In a six-deck game there are 312 cards. If the cut card stops play around 75 percent of the shoe, the count is read only until roughly 234 cards have been dealt. At the next card the shoe is not studied further; it is reshuffled and the function starts over. From a player's side of the table, the cut card is a domain wall.

The count also reminds us that some domains are discrete. You do not read the running count after 12.6 cards; you read it after 12 cards or after 13. A graph may draw those values smoothly, but the real shoe advances in whole cards. Later, when calculus gives us smooth approximations, this difference will matter.

Reading a domain is therefore a small investigation. Ask first what the situation permits, then ask what the arithmetic permits. The domain is whatever survives both questions.

This is the first bridge to limits. When a rule forbids an input, calculus often asks a sharper question: not "what happens at the forbidden point?" but "what value is the rule heading toward as the input gets close?"

PRACTICE

STATE A table takes bets from 5 to 200 chips and pays 3-to-2. State the domain and range of p(b) = 1.5b.

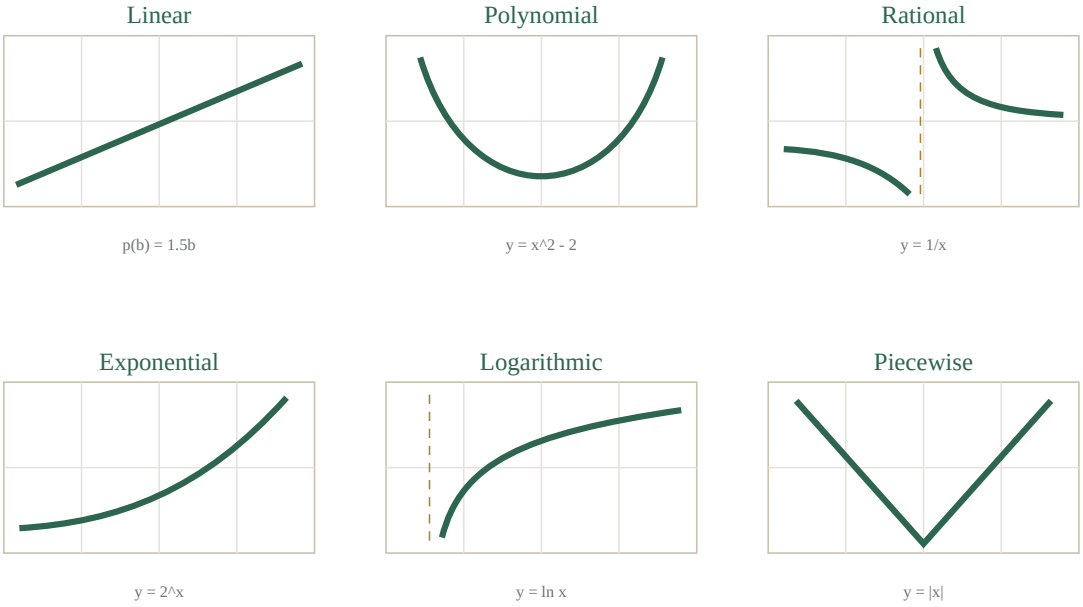
FIND Name every forbidden input: 12/(b - 3) and sqrt(b - 4).

EXPLAIN Can the same formula with two different domains be two different functions?

LESSON 3

The Function Families

Not every rule is a straight line. Functions come in recognizable families, and a family tells you what kind of behavior to expect: steady growth, turning points, forbidden inputs, rapid multiplication, slow growth, or rule changes at boundaries.



The working cast: six families, six characteristic shapes.

Read the families as behaviors. A linear rule changes at a constant rate. A polynomial can turn around. A rational rule is a ratio, so it often forbids inputs. An exponential multiplies rather than adds. A logarithm grows, but ever more slowly. A piecewise rule changes instructions at boundary points.

A SEVENTH TABLE SHAPE

The running count is a step function. It stays flat between cards, then jumps the instant a card is turned. That matters because the real game is discrete and chunky, while calculus studies smooth curves that model it. Good modeling starts by admitting that difference.

- **TABLE MOMENT** *The count sits still while the dealer reaches for the shoe. One card turns, and the count jumps. A graph may look smooth later; the shoe deals in whole cards.*

A doubling-up betting system is another useful warning. The bets 1, 2, 4, 8, 16, 32 form exponential growth. That is why the system fails in practice: the required bet outruns any table maximum and any real bankroll. Changing bet size does not change the expected value of a game already tilted against you.

Away from the table, these families are the vocabulary of applied mathematics: linear models for steady rates, exponentials for growth by percent, logarithms for scales such as decibels and pH, rationals for ratios and concentrations, and polynomials for curves that bend and turn. Learn the six shapes once and you will keep seeing them.

Each family also behaves in its own way under calculus. A polynomial's slope follows a pattern. An exponential's rate is tied to its own size. A rational function's forbidden inputs forecast limits. Naming the family is often the fastest way to predict what kind of calculus is coming.

PRACTICE

IDENTIFY Name the family: 7 - 2x, 5*2^x, (x + 1)/(x - 4), and x^3 - x.

MATCH Which family often has a vertical wall it cannot cross?

SKETCH Draw y = |x - 2| and name the location of its corner.

LESSON 4

Reading the Notation

The symbol *f(x)* is not multiplication. It means "the function *f*, evaluated at input *x*." Whatever sits inside the parentheses is the input. You place that input everywhere the variable appears.

- **TABLE MOMENT** *If S(5) reports your stack after five hands, the 5 is not multiplying anything. It is the moment you ask the rule to read the stack.*

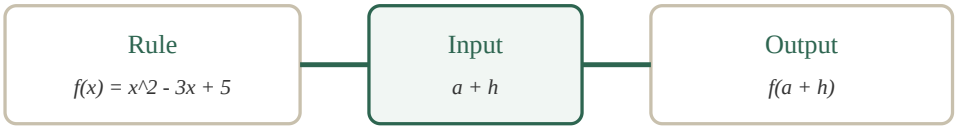
If *f(x) = x^2 - 3x + 5*, then substitution gives:

Input	Substitute	Output
f(2)	2^2 - 3*2 + 5	3
f(0)	0 - 0 + 5	5
f(-1)	1 + 3 + 5	9

The input need not be a number. It can be an expression. If the input is *a + h*, then

$$f(a + h) = (a + h)^2 - 3(a + h) + 5$$

This is ordinary substitution, but it is the raw material of calculus. The difference *f(a + h) - f(a)*, divided by *h*, measures how much the output moved per unit of input. That expression becomes the derivative.



replace every x with the whole expression a + h

Function notation is a substitution instruction, not multiplication.

PRACTICE

COMPUTE For g(x) = 2x + 7, find g(3), g(0), and g(-4).

SUBSTITUTE For f(x) = x^2, write and simplify f(a + h).

READ In one sentence, say what f(5) means for a function that reports chips won after five hands.

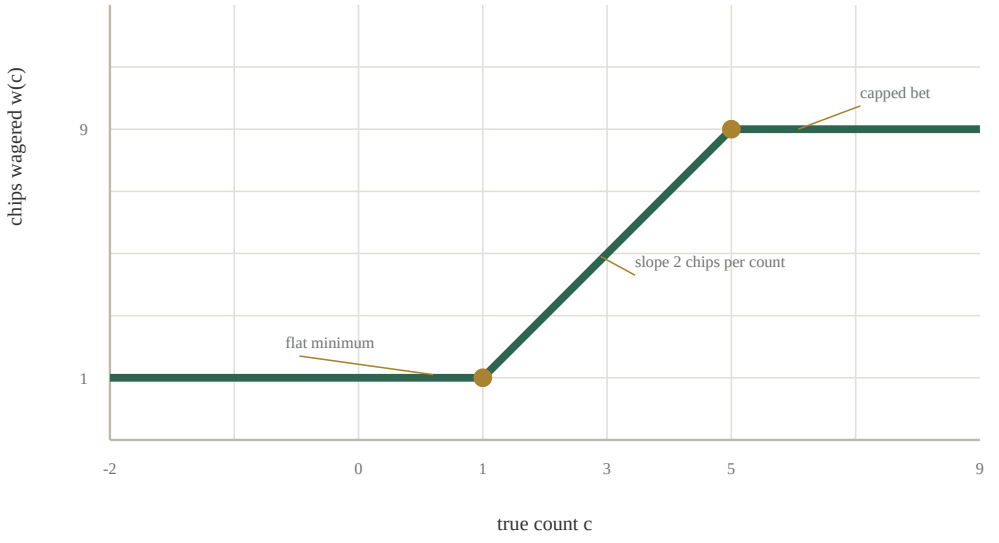
LESSON 5

The Shape of a Rule

A formula tells you the rule. A graph lets you see behavior: where a function holds steady, where it rises, where it stops, and where the interesting changes happen.

Here is a card counter's betting plan drawn as a function. The input is the true count *c*, a measure of how rich the remaining cards are in high cards. The output is the wager *w(c)*, measured in chips.

- **TABLE MOMENT** *The count climbs past +1. Your hand goes to the chip tray and comes back heavier. The ramp just moved.*

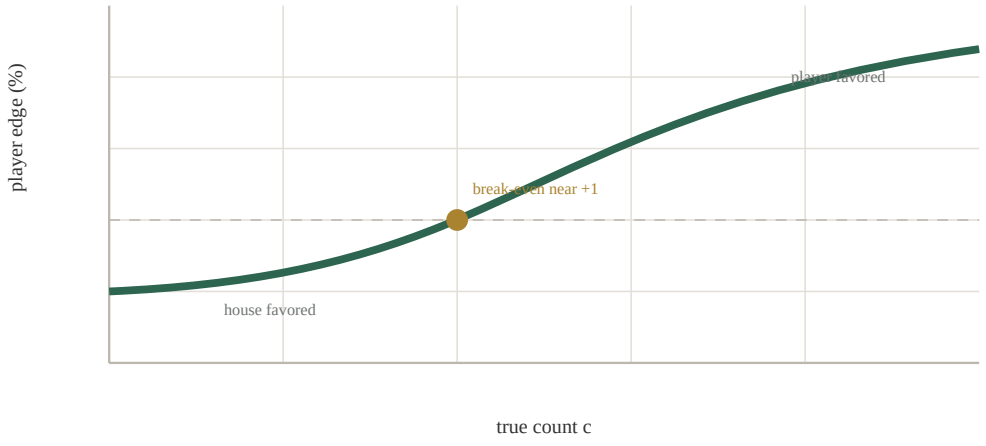


A betting ramp is one function with three behaviors: flat, rising, capped.

Read it left to right. While the count is 1 or below, the wager stays at one chip. Between counts 1 and 5, every extra point of count adds 2 chips. After count 5, the bet is capped at 9 chips. The interesting mathematics lives where the behavior changes.

THE EDGE BEHIND THE RAMP

The ramp is the response. The opportunity behind it is the player's edge. A simple model puts the edge slightly negative at count 0, near break-even around +1, and positive after that. The ramp begins climbing only after the opportunity turns.



The edge curve explains the ramp: bet small when negative, increase when positive.

Slope is the rate of change of a straight piece. The derivative, coming later, is how calculus talks about slope when the graph curves and the rate changes from point to point.

PRACTICE

READ From the ramp, what is w(0)? w(3)? w(7)?

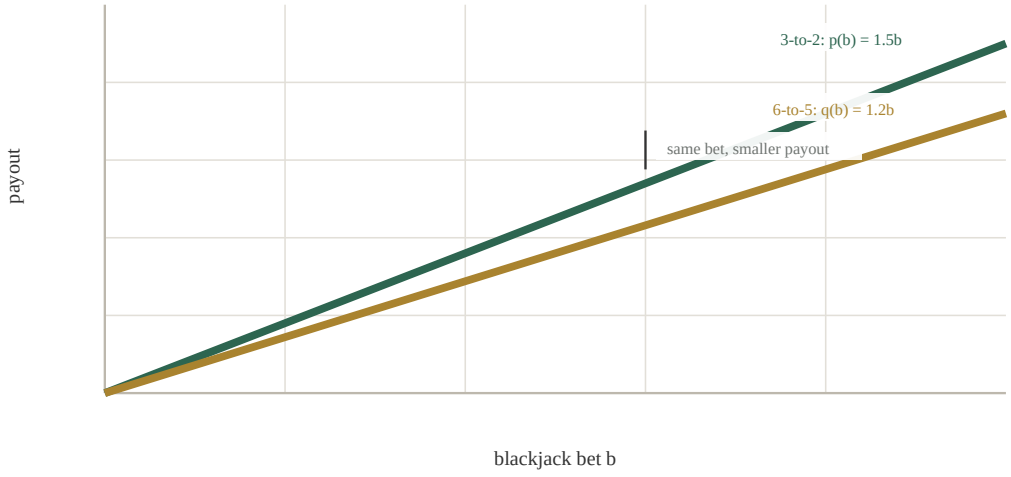
INTERPRET Where is the slope zero, and what does zero slope mean for the bettor?

CRITIQUE A friend says "a bigger count always means a bigger bet." Which part of the graph proves that false?

LESSON 6

Changing the Rule

When a casino changes a blackjack payout from 3-to-2 to 6-to-5, it is not inventing new mathematics. It is transforming a function you already know. The original rule is $p(b) = 1.5b$. The 6-to-5 rule is $q(b) = 1.2b$.



Dropping from 3-to-2 to 6-to-5 compresses the payout line.

Read straight up from any bet. The gold line sits below the green line, and the gap widens as the bet grows. That visible gap is the lost payout. This is why players avoid 6-to-5 blackjack: the slope changed.

Scaling is one transformation. Adding a constant shifts a graph up or down. Replacing the input with $b - k$ slides a graph left or right. Multiplying by a negative flips it. These moves let you predict a graph before you plot it.

The practical point is that a small-looking formula change can be a large table change. At 40 chips, the 3-to-2 payout is 60 chips and the 6-to-5 payout is 48 chips. The difference is 12 chips on one natural blackjack. The graph shows this without any speech: lower slope means less return for every bet.

- **TABLE MOMENT** *Same 40-chip natural. At 3-to-2 you get 60. At 6-to-5 you get 48. Twelve chips walked away because the slope changed.*

THE SAME RAMP, NEW TRIGGER

A cautious counter might keep the ramp shape but wait until the true count reaches +2 before increasing the bet. That is a horizontal shift. A larger-bankroll player might use the same trigger but multiply every wager by 4. That is a vertical stretch. Same family, different stakes, different risk.

This is how transformations become strategy language. A vertical stretch changes bet size. A horizontal shift changes the trigger. A cap cuts off the graph at the top. A floor keeps the graph from dropping below the table minimum. Once you can see those moves, a betting system stops being mysterious and becomes a piecewise function with choices attached.

None of these transformations changes the underlying house rules. They change the player's exposure to the rule. That distinction matters all through the book: a function can model a decision without pretending the decision changes the game underneath it.

PRACTICE

PREDICT Describe how $p(b) = 1.5b + 4$ differs from $p(b) = 1.5b$.

COMPARE At a 40-chip bet, how many chips does a player lose by sitting at 6-to-5 instead of 3-to-2?

NAME What transformation turns $p(b) = 1.5b$ into $q(b) = -1.5b$?

LESSON 7

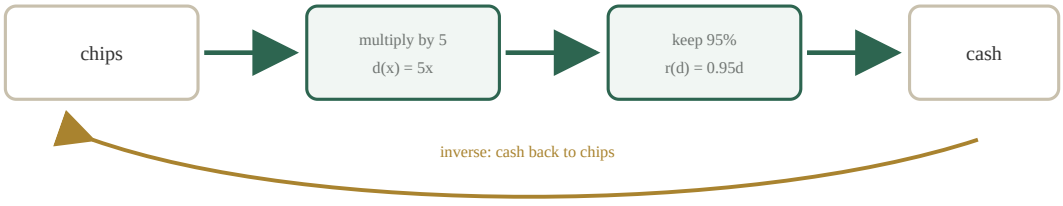
Chaining and Undoing Rules

Real quantities usually pass through more than one rule. Functions handle this with **composition**, which feeds the output of one rule into the next, and with **inverses**, which run a rule backward.

Cash out a stack of chips. First, each chip is worth 5 dollars: $d(x) = 5x$. Then the room takes a 5 percent fee: $r(d) = 0.95d$. The composition is

- **TABLE MOMENT** *The cage counts chips, converts them to dollars, then the fee comes off. The order is not decoration; it is the chain.*

$$r(d(x)) = 0.95(5x) = 4.75x$$



Composition moves through a chain; an inverse walks the chain backward.

The inverse answers the reverse question: "If I walked out with 95 dollars, how many chips did I cash?" Since the single collapsed rule is $f(x) = 4.75x$, the inverse is $f^{-1}(y) = y/4.75$. For 95 dollars, the answer is 20 chips.

Order matters. Converting chips and then taking a fee is not the same as applying a fee to chips and then converting, unless the fee rule has been defined very carefully. In composition, the inside function happens first. That is why the notation $r(d(x))$ is read from the inside out.

COUNT TO EDGE TO BET TO WIN

A player's long-run result is a chain. The cards dealt determine the count. The count helps estimate the edge. The edge triggers a bet. The bet and edge together determine expected win. Written compactly, that is

$$W(B(E(C)))$$

Chapter 5 will give this chain its calculus rule. For now, the lesson is simpler: big models are often small functions stacked in the correct order.

A USEFUL INVERSE: THE BUST POINT

Recall the drain model $B(h) = 100 - 2h$. Turn the question around. Instead of asking "how many chips after h hands?" ask "how many hands until the stack reaches a chosen chip count?" Solving for h gives $h(B) = (100 - B)/2$. Set $B = 0$, and the inverse returns $h = 50$: the bust point of the simplified model.

Inverses are powerful because players often ask reverse questions. How many hands until broke? How many chips created this cash-out? What true count would justify a target bet? Each question runs a function backward, if the function is one-to-one enough to allow it.

PRACTICE

CHAIN With d(x) = 5x and r(d) = 0.95d, compute the take-home dollars for 60 chips.

INVERT Write the inverse of f(x) = 4.75x and use it to find how many chips produced 200 dollars.

JUDGE The rule "hand total from the cards" has no inverse. Explain why.

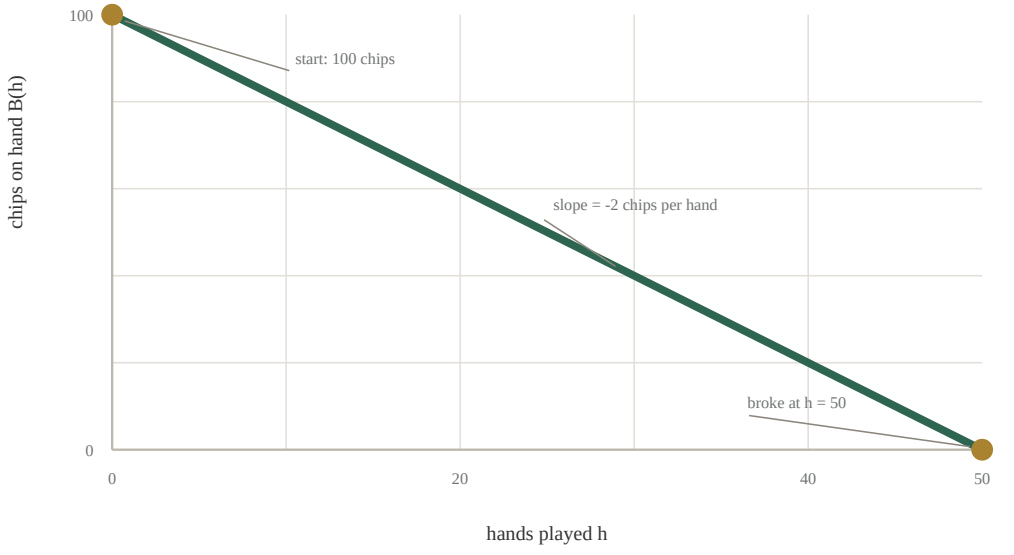
MODEL LAB

Building a Model from Scratch

A lesson shows finished functions. A model is what you get when you build one: turning a loose situation into a rule precise enough to compute with.

Suppose a player sits down with 100 chips and, on flat bets, drifts down about 2 chips per hand. The input is hands played, *h*. The output is chips on hand, *B*. The domain is whole hands from 0 until the stack reaches zero. The simplified model is

$$B(h) = 100 - 2h$$



A good model labels the intercept, slope, and edge of the domain.

Two checks make the model trustworthy. An exact calculation: after 20 hands, $B(20) = 100 - 40 = 60$ chips. A reasonableness check: the model stops at zero because a player cannot lose chips they no longer have.

A third check is units. The starting value is measured in chips. The slope is chips per hand. Multiplying *2 chips per hand* by *h hands* leaves chips, which can be subtracted from 100 chips. When units do not agree, the formula is telling you something is wrong before the arithmetic even starts.

- **TABLE MOMENT** *You just won 50 chips in five hands, but the model says you should be down 10. The function describes the average pull of the game, not the bounce of one shuffle.*

THE CASINO MODELS YOU TOO

While your stack changes, the house has its own function. It estimates theoretical loss and returns a fraction as comps. A simple comp model might be $K(h) = 0.1h$, comp value in chips after h hands. Your stack model slopes down. The comp model slopes up. Two functions, two points of view.

This is not a claim that any single player will lose exactly 2 chips per hand. Sessions swing. A player can win for hours inside a negative-expectation game. The model is an average story, not a fortune-telling machine. A good model is useful precisely because it says what it is leaving out.

YOUR TURN

Build a one-variable model for a situation of your choice. Give five things: a formula, a labeled sketch, one exact calculation, one numerical check, and one honest sentence about where the model breaks.

TRANSFER LAB

Away From the Table

The felt was a memory aid. If the mathematics belongs to you, it also works without the theme. Work these with symbolic steps first, then check one numerical case.

POLYNOMIAL For f(x) = x^2 - 3x + 5, state the domain, then form and simplify [f(a + h) - f(a)]/h.

RATIONAL For g(x) = 1/(x - 2), state the domain and find the inverse function.

EXPONENTIAL For k(x) = 2*3^x, evaluate k(0) and k(2), then describe the transformation from 3^x.

PIECEWISE A garage charges 3 for the first hour, 2 per hour for hours 2 through 5, and a flat 12 after 5 hours. Write C(t), state the domain, and sketch it.

SELF-CHECK

Four honest questions close the chapter. Can I name the input and output before I write a formula? Can I state the domain before trusting a graph? Can I spot an impossible sign or impossible unit? Can I solve the same structure when the blackjack story is stripped away?

If the answer is yes, the chapter has done its job. If one answer is no, that is not failure; it is a map. Go back to the lesson where the question first appeared and rebuild that one piece.

UNITS If the slope is -2 chips per hand and you play 10 hands, the model predicts -20 chips of change. If your formula says -200, what unit or multiplication error should you suspect?

CHAPTER 1 REVIEW

Pulling It Together

This is not a pile of look-alike problems. It is a tour back through the chapter's seven ideas, in the order you met them. If one prompt stalls you, that is the lesson to reread.

CHAPTER REVIEW

DEFINE In one precise sentence, say what makes a rule a function.

JUDGE Give one pairing from ordinary life that is not a function, and name the input that breaks it.

APPLY Using EV(p) = 3p - 1, state the ten-density at which insurance breaks even.

DOMAIN State the domain of 12/(x - 3) and explain the one forbidden input.

RANGE A 3-to-2 table takes 5- to 200-chip bets. Give the range of p(b) = 1.5b.

IDENTIFY Name the family of each rule: 7 - 2x, 5*2^x, and (x + 1)/(x - 4).

SPOT A student writes f(a + h) = f(a) + f(h). Give one function that proves this false.

READ On a betting ramp, what does a flat stretch tell you?

PREDICT Describe, without plotting, how p(b) = 1.5b + 4 differs from p(b) = 1.5b.

SUMMARIZE In exactly three sentences, say what a function is, why its domain matters, and how composition builds bigger rules from small ones.

LOOKING AHEAD

From Rules to Change

LOOKING AHEAD

You now have the vocabulary the rest of the book speaks: a function is a reliable rule; its domain is what it accepts; its range is what it returns; its graph shows behavior; its slope shows rate of change; transformations reshape it; and composition chains rules together.

Every idea quietly points toward the same next question: as the input creeps closer and closer to a value, where is the output going? That question is called a *limit*, and it is where calculus begins in earnest.

Limits are the right next step because the table keeps handing us boundary questions. What happens near the bet minimum? What happens as a denominator gets close to zero? What happens as the count approaches a trigger point from below? Chapter 2 turns those "near" questions into precise mathematics.

The important shift is this: functions tell us the rule, but calculus asks how the rule behaves as we move. Chapter 1 built the board. Chapter 2 starts watching the motion.